Application and Comparison of Regression and Markov Chain Methods in Bridge Condition Prediction and System Benefit Optimization

by Yi Jiang

The maximization of a bridge system is achieved using mathematical optimization techniques, such as linear programming and dynamic programming. For each bridge, the input data of the bridge project selection model includes the predicted bridge condition in future years, the recommended bridge repair action, the estimated cost of recommended bridge repair action, and the expected improvement or benefit from the repair action. Through mathematical manipulation, bridge projects are selected to maximize the total expected benefit of the bridge system while a number of constraints are simultaneously satisfied. This optimization process is based on the predicted bridge conditions. Therefore, the accuracy of bridge condition predictions is vital to the effectiveness of bridge project selection. This paper shows that bridge condition predictions will affect bridge project selections and the corresponding system benefits.

INTRODUCTION

The ultimate objective of a bridge management system is to select bridge projects for a multiyear period so that the total system benefit will be maximized through performing the scheduled maintenance, rehabilitation, and replacement activities to the selected bridges. In a bridge management system, the maximization of a bridge system benefit is achieved using mathematical optimization techniques, such as integer linear programming and dynamic programming (Winston 2003). For each bridge, the input data of the bridge project selection model include the predicted bridge condition in future years, the recommended bridge repair action, the estimated cost of recommended bridge repair action, and the expected improvement or benefit from the repair action. Through mathematical manipulation, bridge projects are selected to maximize the total expected benefit of the bridge system while a number of constraints are simultaneously satisfied. Bridge condition rating is the most important variable considered in the process of bridge project selection. Decision making, either at the system level or at the project level, is based on bridge conditions at present and in the future. The accuracy of the future condition prediction directly affects the outcome of optimization in selecting bridge projects. Therefore, the accuracy of bridge condition predictions is vital to the effectiveness of bridge project selection. If the predicted bridge conditions are not accurate, the selected bridge projects will not result in a truly maximized system benefit.

The purpose of this study was to help highway engineers and planners identify and choose an appropriate method for bridge condition predictions. The statistical regression theory (Neter et al. 1985) and the Markov chain theory (Winston 2003) have been applied to predict structural conditions in bridge management systems. In this paper, the accuracies of condition predictions based on the two theories are compared with the Indiana highway bridge condition data. Under a limited budget for bridge repair and rehabilitation, bridge projects can be selected from a given group of bridge candidates through mathematical optimization so that a maximum system benefit can be achieved. A key requirement for the optimization in bridge project selection is the ability to obtain reliable bridge condition predictions. As the available budget decreases, fewer bridges will be selected for repair and rehabilitation. Less bridge repair or rehabilitation now will lead to higher cost in the future because of further bridge condition deteriorations.

CONDITION PREDICTION MODELS

Regression Methods

There are various types of bridge condition prediction models based on statistical regression theory. Applications of piecewise linear regression in bridge condition prediction can be found in Fitzpatrick et al. (1981) and Hymon et al. (1983). Linear regression with two independent variables, bridge age and average daily traffic (ADT), was applied to predict the conditions of bridge deck, superstructure, and substructure (Busa et al. 1985).

According to the FHWA bridge rating system, bridge inspectors use a range from 0 to 9, with 9 being the maximum rating number for an excellent condition and 0 being the rating for a failed and out of service condition (USDOT 2006). The objective of developing regression equations was to find the relationship between condition rating and bridge age. The polynomial regression method was applied to predict bridge conditions in Indiana (Jiang and Sinha 1989). A third order polynomial model was used to obtain the regression function of the relationship. The polynomial model is expressed by the following formula (Neter et al. 1985).

(1) $$Y(T_i) = \beta_0 + \beta_1 T_i + \beta_2 T_i^2 + \beta_3 T_i^3 + \varepsilon_i$$

Where:

$Y(T_i)$ – Condition rating of bridge i, $0 \leq R \leq 9$, with rating 9 as the rating of a perfect condition;
T_i – Age of bridge i;
β_0, β_1, β_2, β_3 – Regression coefficients;
ε_i – Error term.

The Indiana inspection includes ratings of individual components such as deck, superstructure, and substructure as well as of the overall bridge condition. The complete data base included about 5,700 state owned bridges in Indiana. Through statistical analysis and regression, the regression equations were developed for concrete and steel bridges and bridge components (deck, superstructure, and substructure) on Indiana interstate highways and non-interstate highways (Jiang and Sinha 1989). As examples, some of the developed regression equations developed for Indiana bridges are listed below.

- Deck conditions of steel bridges on interstate highways:
$$Y(T_i) = 9 - 0.41141790\ T_i + 0.02116563\ T_i^2 - 0.00040387\ T_i^3$$

- Superstructure conditions of concrete bridges on non-interstate highways:
$$Y(T_i) = 9 - 0.29095931\ T_i + 0.00860726\ T_i^2 - 0.00008815\ T_i^3$$

- Substructure conditions of concrete bridges on interstate highways:
$$Y(T_i) = 9 - 0.34508455\ T_i + 0.01575857\ T_i^2 - 0.00026681\ T_i^3$$

As can be seen, the condition rating $Y(T_i)$ can be predicted based on the bridge age with the regression equation once the bridge type and highway type are identified.

Markov Chain Approach

The Markov chain is a special case of stochastic processes (Winston 2003). The theory of stochastic processes has been applied in many areas of engineering and other applied science. For example, the theory was used by Li and Zhang (2007) for soil mapping from irregularly distributed point samples.

Caleyo et al. (2009) developed an empirical Markov chain-based stochastic model for predicting the evolution of pipeline pitting corrosion depth and rate distributions from the observed properties of the soil. The Markov model was also applied to analyze the impact of Wal-Mart on the grocery market and to develop the competitive strategies of grocery retailers (Yang et al. 2010).

A stochastic process is said to be Markovian if given the value X(t), the value of X(s) for s > t does not depend on the value of X(μ) for μ < t. In other words, the future behavior of the process depends only on the present state but not on the past. In formal terms, a process is said to be Markovian if

$$(2)\quad P[a < X(t) \leq b / X(t_0) = x_0, \ldots, X(t_n) = x_n] = P[a < X(t) \leq b / X(t_n) = x_n]$$

where $t_0 < t_1 < \ldots < t_n < t$.

The theory was applied in pavement management systems (Butt et al. 1987, Li et al. 1996), storm water pipe deterioration modeling (Micevski et al. 2002), and bridge management systems (Jiang and Sinha 1989). Essentially, a stochastic process is a probability-based process describing the changes of random variables in time. The Markov chain as applied to bridge performance prediction is based on the concept of defining states in terms of bridge condition ratings and obtaining the probabilities of bridge condition changing from one state to another. These probabilities are represented in a matrix form that is called the transition probability matrix or simply, transition matrix, of the Markov chain. Knowing the present state of bridges, or the initial state, the future conditions can be predicted through multiplications of the initial state vector and the transition probability matrix.

Ten bridge condition ratings are defined as 10 states with each condition rating corresponding to one of the states. For example, condition rating 9 is defined as state 1, rating 8 as state 2, and so on. Without repair or rehabilitation, the bridge condition rating decreases as the bridge age increases. Therefore, there is a probability of condition changing from one state, say i, to another state, j, during a given period of time, which is denoted by $p_{i,j}$.

$$(3)\qquad P = \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} & \cdots & \cdots & \cdots & p_{1,10} \\ 0 & p_{2,2} & p_{2,3} & \cdots & \cdots & \cdots & p_{2,10} \\ 0 & 0 & p_{3,3} & \cdots & \cdots & \cdots & p_{3,10} \\ 0 & 0 & 0 & p_{4,4} & \cdots & \cdots & p_{4,10} \\ 0 & 0 & 0 & 0 & p_{5,5} & \cdots & p_{5,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & p_{10,10} \end{bmatrix}$$

In the transition matrix, $p_{1,1}$ is the probability of condition changing from state 1 (rating 9) to state 1 (rating 9) in one year, $p_{1,2}$ from state 1 (rating 9) to state 2 (rating 8), and so on. As shown in the transition matrix, some of the transition probabilities are equal to 0. This is because the bridge condition ratings will not increase without repair or rehabilitation actions. To simplify the transition matrix, Jiang and Sinha (1989) made some realistic assumptions according to actual bridge condition data. First, it is assumed that the bridge condition rating would not drop by more than one in a single year. This is reasonable because the bridge condition rating in Indiana seldom drops more than one in a single year as found by the research team that developed the Indiana Bridge Management System (Sinha et al. 1989). Second, it is assumed that the lowest bridge condition rating is 3, because it is an FHWA requirement that a bridge be repaired or replaced when its condition rating reaches 3. Any bridge with a condition rating below 3 must be closed due to safety concerns. With the two assumptions, the transition matrix of condition ratings has the following form:

(4)

$$P = \begin{bmatrix} p(1) & q(1) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2) & q(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3) & q(3) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4) & q(4) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5) & q(5) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6) & q(6) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(4)

where:

p(i) – Transition probability from state i to state i;

q(i) – Transition probability from state i to state i-1, q(i)=1-p(i).

In the matrix, p(1) is the transition probability from rating 9 (state 1) to rating 9, and q(1), from rating 9 to rating 8 (state 2), and so on. It should be noted that the lowest rating number before a bridge is repaired or replaced is 3. Consequently, the corresponding transition probability p(7) equals 1.

Knowing the present condition of a bridge, or the initial state, the future conditions can be predicted through multiplications of initial state vector $Q_{(0)}$ and the transition matrix P. The state vector for year T, $Q_{(T)}$, can be obtained by the multiplication of initial state vector $Q_{(0)}$ and the Tth power of the transition probability matrix P:

(5) $Q_{(T)} = Q_{(0)}\, P\, P \ldots P = Q_{(0)}\, P^T$

Equation 5 is equivalent to the following:

(6) $Q_{(T)} = Q_{(T-1)}\, P$

Thus, a Markov chain is completely specified when its transition matrix P and the initial state vector $Q_{(0)}$ are known. Since the initial state vector $Q_{(0)}$ is usually known, the main problem of the Markov chain approach in this study is to determine the transition probability matrix. The detailed description of the transition probability matrix development is given in Jiang and Sinha (1989).

The maximum rating of bridge condition is 9 and it represents a near-perfect condition. It is almost always true that a new bridge has condition rating 9. In other words, a bridge at age 0 has condition rating 9 with unit probability. Thus, the initial state vector $Q_{(0)}$ of a new bridge is always [1, 0, 0, 0, 0, 0, 0], where the numbers are the probabilities of having condition rating of 9, 8, 7, 6, 5, 4, and 3 at age 0, respectively.

An essential property of Markov chain is that the future behavior of the process depends only on the present state but not on the past. As long as the bridge condition is known at any time, the state vector at that time can be used as the initial vector $Q_{(0)}$ to predict the future condition. For example, if a bridge is 10 years old with a condition rating 7, then the initial state vector $Q_{(0)}$ should be [0, 0, 1, 0, 0, 0, 0]. That is, the unit probability corresponds to condition rating 7 and the current time (age 10) is used as the starting time (time 0). With this initial state vector and a transition probability matrix, future condition ratings of this bridge can be estimated from age 10.

Let R be a vector of condition ratings, R=[9 8 7 6 5 4 3], and R' be the transform of R, i.e.,

$$R' = \begin{vmatrix} 9 \\ 8 \\ 7 \\ 6 \\ 5 \\ 4 \end{vmatrix}$$

Then the estimated condition rating at year t by Markov chain is

(7) $E(t) = Q_{(t)} R'$

Equation 7 can also be expressed as:

(8) $E(t) = Q_{(0)} P^t R'$

An example set of computations is given in the following. The transition matrix for deck conditions of concrete bridges on non-interstate highways was obtained (Jiang and Sinha 1989):

(9) $$P = \begin{vmatrix} 0.700 & 0.300 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.780 & 0.220 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.874 & 0.126 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.600 & 0.400 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.500 & 0.500 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.400 & 0.600 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{vmatrix}$$

For illustration, p(1)=0.700 indicates that the probability of bridge deck condition changing from state 1 (condition rating 9) to state 1 (remaining in state 1) in a one-year period is 0.700, and the probability of changing from state 1 to state 2 (condition rating 8) is q(1)=0.300. Similarly, p(2)=0.780 indicates that the probability of transitioning from state 2 to state 2 (remaining in state 2) in a one-year period is 0.780, and the probability of transitioning from state 2 to state 3 (condition rating 7) is q(2)=0.220.

Assuming there is a new concrete bridge with a condition rating 9, the initial state vector should be $Q_{(0)}$ = [1 0 0 0 0 0 0]. The bridge deck's condition rating can be predicted by Equations 6 and 7 with the matrix P (Equation 9). For example, the state vectors and condition ratings for year 0 through year 6 are given as follows:

$R = [9\ 8\ 7\ 6\ 5\ 4\ 3]$

$Q_{(0)} = [1\ 0\ 0\ 0\ 0\ 0\ 0]$
$E(0) = Q_{(0)}\ R' = 9.0$

$Q_{(1)} = Q_{(0)}\ P = [0.70\ 0.30\ 0.00\ 0.00\ 0.00\ 0.00\ 0.00]$
$E(1) = Q_{(1)}\ R' = 8.70$

$Q_{(2)} = Q_{(1)}\ P = [0.49\ 0.44\ 0.07\ 0.00\ 0.00\ 0.00\ 0.00]$
$E(2) = Q_{(2)}\ R' = 8.42$

$Q_{(3)} = Q_{(2)}\ P = [0.34\ 0.49\ 0.16\ 0.01\ 0.00\ 0.00\ 0.00]$
$E(3) = Q_{(3)}\ R' = 8.17$

$Q_{(4)} = Q_{(3)}\ P = [0.24\ 0.49\ 0.24\ 0.03\ 0.00\ 0.00\ 0.00]$
$E(4) = Q_{(4)}\ R' = 7.94$

$Q_{(5)} = Q_{(4)}\ P = [0.17\ 0.45\ 0.32\ 0.05\ 0.01\ 0.00\ 0.00]$
$E(5) = Q_{(5)}\ R' = 7.72$

$Q_{(6)} = Q_{(5)}\ P = [0.12\ 0.40\ 0.38\ 0.07\ 0.02\ 0.01\ 0.00]$
$E(6) = Q_{(6)}\ R' = 7.50$

The above example used age 0 as the starting time. However, it should be pointed out that the Markov prediction can be performed using any point in time as the starting time as long as the condition rating is known. This is because the future behavior of the Markov process depends only on the present state but not on the past.

Some states have started to use element-level bridge inspections for bridge condition ratings (FHWA 2009). The element-level bridge inspection system was proposed by the American Association of State Highway and Transportation Officials (AASHTO) in 1995 (AASHTO 1995). The element-level inspection divides bridge structural components into sub-elements. Therefore, it contains more bridge elements for inspection and provides more detailed bridge condition information. However, the Federal Highway Administration (FHWA) has found widespread variability in the elements used by states. The lack of uniformity in states' use of element-level data has impeded federal efforts to collect and use element-level bridge data (FHWA 2009). Nonetheless, if it is needed, the prediction methods described in this paper can be readily applied to the element-level bridge data because the general principles remain the same.

BRIDGE PROJECT SELECTION AND SYSTEM BENEFIT OPTIMIZATION

Optimization techniques manipulate the tradeoffs between the objective and constraints systematically or mathematically, so that an optimal solution to the problem among many possible solutions can be obtained. In managing a bridge system, optimization techniques can be applied to produce optimal strategies in project selection by maximizing the system benefit subject to the constraints, such as available resources. An integer linear programming model is used in the following to demonstrate the effects of bridge condition predictions on system benefits. The optimization model is formulized as follows:

Objective function:

$$(10)\ \max \sum_{t=1}^{T} \left(\sum_{i} \sum_{a} X_{i,t} \times E_i \right)$$

Subject to the following constraints:

(a) available budget:

$$(11) \sum_{t=1}^{T} \left(\sum_{i} \sum_{a} X_{i,t} \times c_i \right) \leq B$$

(b) one activity cannot be undertaken more than once on one bridge in T years:

$$(12) \sum_{t=1}^{T} X_{i,t} \leq 1$$

(c) zero-one integer decision variable:

$$(13)\ X_{i,t} = 0 \text{ or } 1$$

where:

$X_{i,t}$	=	1, if bridge i is chosen for the proposed rehabilitation or replacement;
$X_{i,t}$	=	0, otherwise;
E_i	=	effectiveness gained by bridge i if the proposed rehabilitation or replacement activity is conducted;
B	=	total available budget for the program period;
c_i	=	estimated cost of activity a on bridge i;

The effectiveness of a bridge improvement activity is defined as follows:

$$(14)\ E_i = ADT_i \times \Delta A_i \times (1+Csafe_i) \times (1+Cimpc_i)$$

where:

ADT_i	=	average daily traffic on bridge i.
ΔA_i	=	area under regression curves of bridge i obtained by the proposed rehabilitation activity; as shown in Figure 1.
C_{safei}	=	coefficient of safety condition of bridge i, converted from bridge safety utility value; as shown in Figure 2.
C_{impci}	=	coefficient of community impact of bridge i, converted from community impact utility value in terms of detour length; as shown in Figure 3.

Figure 1: Condition Improvement by Rehabilitation

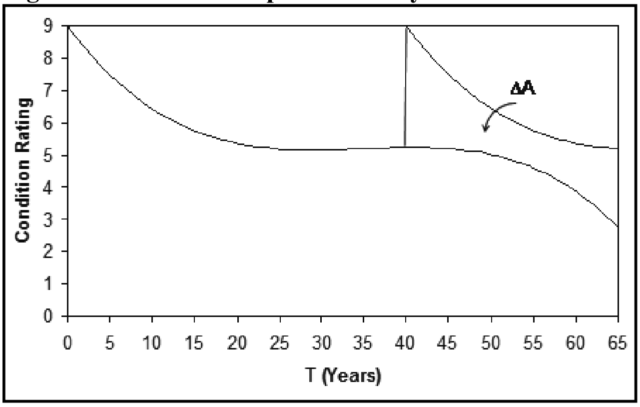


Figure 2: Coefficient of Safety Condition

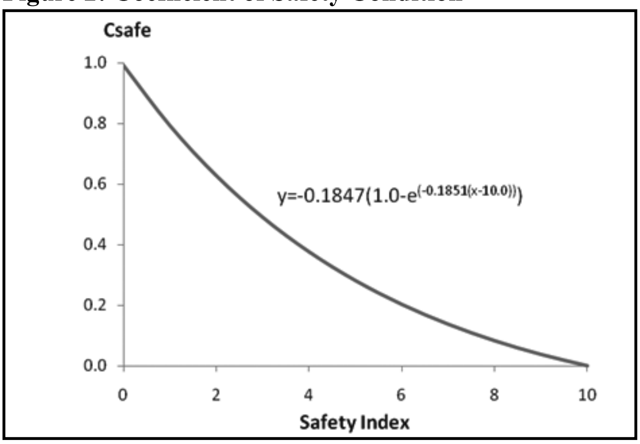
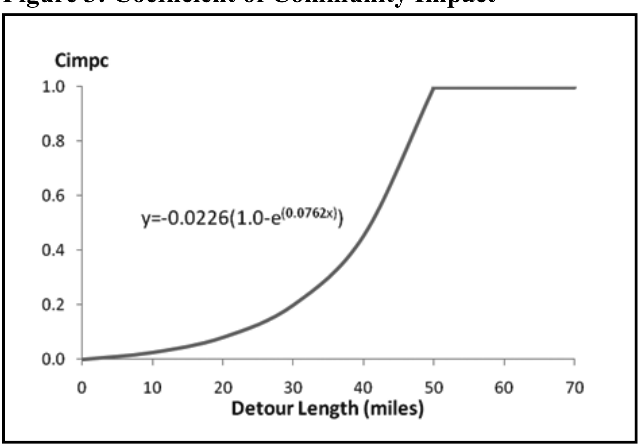


Figure 3: Coefficient of Community Impact



As E_i is defined as the effectiveness gained by bridge i if the proposed rehabilitation or replacement activity is conducted, it is actually the benefit to be realized if bridge i is chosen. As indicated in Figure 1, the benefit begins right after the bridge is repaired or replaced and lasts until the end of the average service life. There are several ways that the effectiveness of a bridge activity can be defined. Because ADT is the number of vehicles served by a bridge, the inclusion of ADT and ΔA in the effectiveness function (Equation 14) can be interpreted as the measure of the improvement that can be experienced by the users or vehicles passing the bridge. Traffic safety condition and community impact of a bridge are two other factors affecting decisions on bridge rehabilitation or replacement activities in addition to structural condition. Bridge safety index and bridge detour length were used as variables reflecting bridge traffic safety and community impact, respectively. The coefficients, C_{safe} and C_{impc}, were used to modify the effectiveness of individual bridge projects depending on site specific impacts.

As shown in Figure 1, a particular rehabilitation activity causes a jump in the bridge condition rating. As the bridge age increases, the condition rating gradually decreases from the new condition rating. The area between the regression curves with and without rehabilitation, ΔA, represents an improvement in terms of condition rating and service life of the bridge. Figure 2 shows the Indiana coefficient of traffic safety index, C_{safe}, ranging from 0.0 to 1.0. The traffic safety index is primarily based on bridge geometrics and it ranges from 1 to 10 with 10 being the index of no potential safety problem (Jiang and Sinha 1989). The coefficient of community impact (C_{impc}) of bridges is shown in Figure 3 and ranges from 0 to 1.0. The community impact coefficient is based on detour length (Jiang and Sinha 1989). As detour length increases the community impact coefficient increases. Therefore, the effectiveness (E_i) gained by a bridge project is the benefit that the motorists (ADT_i) will enjoy through the improved bridge condition (ΔA_i), the enhanced safety ($1+Csafe_i$), and the positive community impact duo to avoided bridge closure ($1+Cimpc_i$).

In Indiana, bridge rehabilitation activities mainly include deck reconstruction and deck replacement. Deck reconstruction work includes shallow and/or full-depth patching of deteriorated deck spots and an overlay of the deck after scarifying the wearing surface. In order to increase bonding between the bridge deck and the overlay materials, the worn and polished deck surface is scarified by grinding to create rough textures. Along with this reconstruction, curbs, railing, and expansion joints are replaced in most cases. Other related work includes guardrails, approach slab reconstruction, approach shoulder reconstruction, and small amounts of substructure repairs. The deck replacement alternative is a more extensive rehabilitation than deck reconstruction. Deck replacement consists of a replacement of the entire deck, including rehabilitation of parts of the superstructure and the top portion of the substructure. The replacement of the entire bridge is considered when reconstruction and rehabilitation cannot adequately correct the existing deficiencies. Thus, bridge rehabilitation and replacement activities were grouped into three options: deck reconstruction, deck replacement, and bridge replacement.

CONDITION RATING PREDICTIONS

Forty bridges were selected from the Indiana's bridge condition database in 2004 to illustrate bridge condition predictions with polynomial regression and Markov chain methods. The Indiana bridge condition database is used for the Indiana Bridge Management System. It should be pointed out that the main function of a bridge management system is to select bridge projects that will provide maximum system benefit under budget constraint. Therefore, it is a decision-making tool at the system level rather than at the project level. The information pertaining to these selected bridges is shown in Table 1. The proposed activity for each bridge in the table is the result of closer field inspection and engineering decision at the project level. The project level decision is used as an input of the system optimization.

As can be seen in Table 1, each of the selected bridges has four consecutive condition ratings. That is, the actual condition ratings of these bridges are known. Since each bridge was observed and rated once every two years, the four ratings represent bridge conditions observed in 1997, 1999, 2001, and 2003. For example, the condition ratings of the first bridge were (6, 6, 6, 5), meaning that the condition rating was 6 in 1997, 6 in 1999, 6 in 2001, and 5 in 2003.

To compare the condition predictions of the two prediction methods, the condition ratings of the 40 bridges were calculated with the polynomial regression method and the Markov chain method. With the polynomial regression method, each bridge condition rating was calculated using the bridge age as the input. With the Markov chain method, the condition rating in 1997 of each bridge was utilized as the "current" condition to predict the conditions in 1999, 2001, and 2003. The actual and predicted condition ratings for the three years are shown in Tables 2 and 3. Table 2 shows the results from the polynomial regression method and Table 3 shows those from the Markov chain method. As shown in the two tables, each condition prediction error was calculated by the predicted rating subtracting its responding actual rating. To compare the two methods, the prediction errors for 1999, 2001, and 2003 are plotted in Figures 4, 5, and 6, respectively. As clearly illustrated in the three figures, the magnitudes of Markov chain prediction errors are smaller than those of polynomial regression predictions for a majority of the 40 bridges. In other words, for most of the 40 bridges the Markov chain predictions are more accurate than the polynomial regression predictions.

As demonstrated in Tables 2 and 3 as well as in Figures 4, 5, and 6, there are positive and negative errors in the condition predictions. The positive errors represent the overestimates and the negative errors are the underestimates of condition ratings. To quantitatively compare the magnitudes of prediction errors, the absolute values of the prediction errors were used to compute the averages and standard deviations. The reason for using absolute values of the prediction errors was to eliminate the effects of negative values on the magnitudes of the averages and standard deviations. As illustrated in Figures 7 and 8, the Markov chain method produced much better condition rating predictions than the polynomial regression method in terms of both average errors and standard deviations.

Table 1: Information Pertaining to Sample Bridges

Bridge Number	Age	ADT (1000)	Detour Length (mile)	Remaining Service Life	Consecutive Condition Ratings	Proposed Activity	Estimated Cost ($1000)
1	16	95	5.6	20	6, 6, 6, 5	DRC	306
2	27	105	1.2	15	8, 8, 7, 6	DRC	359
3	31	156	1.9	20	4, 4, 3, 3	DRC	516
4	15	63	1.2	20	6, 5, 5, 5	DRC	365
5	36	15	6.8	20	5, 4, 4, 3	DRC	139
6	26	155	5.6	20	8, 7, 7, 7	DRC	157
7	30	162	1.2	15	4, 4, 4, 3	DRC	273
8	39	428	8.6	25	5, 5, 4, 3	DRC	1690
9	30	39	3.7	13	6, 6, 6, 6	DRC	334
10	25	195	1.2	10	5, 5, 5, 4	DRC	429
11	16	92	1.2	10	6, 6, 6, 5	DRC	261
12	16	92	1.2	10	5, 5, 5, 4	DRC	261
13	15	491	1.2	30	6, 6, 6, 5	DRC	619
14	24	248	1.2	25	7, 7, 6, 5	DRC	96
15	23	98	1.2	30	6, 5, 5, 5	DRC	200
16	30	54	3.7	20	5, 5, 5, 4	DRC	86
17	23	47	3.7	20	6, 6, 5, 5	DRC	161
18	18	78	3.1	20	6, 6, 5, 4	DRC	155
19	35	554	8.6	20	6, 6, 6, 5	DRC	321
20	30	179	3.1	12	6, 6, 6, 5	DRC	213
21	56	120	16.0	5	3, 3, 3, 3	BRP	6500
22	49	108	14.2	5	4, 4, 4, 3	BRP	8098
23	49	135	4.9	8	5, 5, 5, 4	BRP	2850
24	47	255	9.3	2	4, 4, 4, 4	BRP	4432
25	69	185	5.6	5	5, 4, 4, 4	BRP	1686
26	58	152	5.6	5	5, 5, 5, 4	BRP	826
27	42	11	23.5	9	5, 5, 4, 4	BRP	1092
28	74	11	23.5	5	4, 4, 3, 3	BRP	1092
29	65	144	1.9	8	6, 6, 6, 5	BRP	4100
30	40	14	3.1	8	6, 6, 6, 5	BRP	384
31	60	26	3.7	2	3, 3, 3, 3	BRP	251
32	21	60	6.8	8	6, 6, 5, 5	BRP	501
33	48	318	1.9	2	4, 4, 4, 4	BRP	2042
34	21	12	3.7	1	4, 4, 3, 3	BRP	504
35	24	9	3.7	1	4, 4, 3, 3	BRP	374
36	81	8	3.7	1	3, 3, 3, 3	BRP	1255
37	56	53	3.7	1	4, 4, 3, 3	BRP	2014
38	72	102	1.2	2	3, 3, 3, 3	BRP	546
39	84	6	8.6	4	3, 3, 3, 3	BRP	85
40	50	8	4.9	1	4, 4, 4, 3	BRP	364

Note: DRC = Deck Reconstruction BRP = Bridge Replacement

Table 2: Accuracy of Bridge Condition Predictions by Regression Method

Bridge Number	Actual Rating in 1999 (A)	Predicted Rating in 1999 (B)	Error (B-A)	Actual Rating in 2001 (C)	Predicted Rating in 2001 (D)	Error (D-C)	Actual Rating in 2003 (E)	Predicted Rating in 2003 (F)	Error (F-E)
1	6	6.0	0.0	6	5.9	-0.1	5	5.8	0.8
2	8	5.6	-2.4	7	5.6	-1.4	6	5.6	-0.4
3	4	5.6	1.6	3	5.6	2.6	3	5.6	2.6
4	5	6.0	1.0	5	5.9	0.9	5	5.8	0.8
5	4	5.6	1.6	4	5.5	1.5	3	5.5	2.5
6	7	5.7	-1.3	7	5.6	-1.4	7	5.6	-1.4
7	4	5.6	1.6	4	5.6	1.6	3	5.6	2.6
8	5	5.5	0.5	4	5.5	1.5	3	5.4	2.4
9	6	5.6	-0.4	6	5.6	-0.4	6	5.6	-0.4
10	5	5.7	0.7	5	5.6	0.6	4	5.6	1.6
11	6	6.0	0.0	6	5.9	-0.1	5	5.8	0.8
12	5	6.0	1.0	5	5.9	0.9	4	5.8	1.8
13	6	6.0	0.0	6	5.9	-0.1	5	5.8	0.8
14	7	5.7	-1.3	6	5.7	-0.3	5	5.6	0.6
15	5	5.7	0.7	5	5.7	0.7	5	5.6	0.6
16	5	5.6	0.6	5	5.6	0.6	4	5.6	1.6
17	6	5.7	-0.3	5	5.7	0.7	5	5.6	0.6
18	6	5.9	-0.1	5	5.8	0.8	4	5.7	1.7
19	6	5.6	-0.4	6	5.6	-0.4	5	5.5	0.5
20	6	5.6	-0.4	6	5.6	-0.4	5	5.6	0.6
21	3	3.9	0.9	3	3.5	0.5	3	3.0	0.0
22	4	4.9	0.9	4	4.7	0.7	3	4.4	1.4
23	5	4.9	-0.1	5	4.7	-0.3	4	4.4	0.4
24	4	5.1	1.1	4	4.9	0.9	4	4.7	0.7
25	4	3.0	-1.0	4	3.0	-1.0	4	3.0	-1.0
26	5	3.5	-1.5	5	3.0	-2.0	4	3.0	-1.0
27	5	5.4	0.4	4	5.3	1.3	4	5.2	1.2
28	4	3.0	-1.0	3	3.0	0.0	3	3.0	0.0
29	6	3.0	-3.0	6	3.0	-3.0	5	3.0	-2.0
30	6	5.5	-0.5	6	5.4	-0.6	5	5.3	0.3
31	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
32	6	5.7	-0.3	5	5.7	0.7	5	5.7	0.7
33	4	5.0	1.0	4	4.8	0.8	4	4.6	0.6
34	4	5.7	1.7	3	5.7	2.7	3	5.7	2.7
35	4	5.7	1.7	3	5.7	2.7	3	5.6	2.6
36	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
37	4	3.9	-0.1	3	3.5	0.5	3	3.0	0.0
38	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
39	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
40	4	4.8	0.8	4	4.6	0.6	3	4.3	1.3

Table 3: Accuracy of Bridge Condition Predictions by Markov Method

Bridge Number	Actual Rating in 1999 (A)	Predicted Rating in 1999 (B)	Error (B-A)	Actual Rating in 2001 (C)	Predicted Rating in 2001 (D)	Error (D-C)	Actual Rating in 2003 (E)	Predicted Rating in 2003 (F)	Error (F-E)
1	6	5.7	-0.3	6	5.4	-0.6	5	5.1	0.1
2	8	7.7	-0.3	7	7.4	0.4	6	7.2	1.2
3	4	3.6	-0.4	3	3.1	0.1	3	3.0	0.0
4	5	5.7	0.7	5	5.4	0.4	5	5.1	0.1
5	4	4.7	0.7	4	4.0	0.0	3	3.4	0.4
6	7	7.7	0.7	7	7.4	0.4	7	7.2	0.2
7	4	3.6	-0.4	4	3.3	-0.7	3	3.0	0.0
8	5	4.7	-0.3	4	4.1	0.1	3	3.3	0.3
9	6	5.7	-0.3	6	5.4	-0.6	6	5.1	-0.9
10	5	4.7	-0.3	5	4.4	-0.6	4	4.1	0.1
11	6	5.6	-0.4	6	5.2	-0.8	5	4.8	-0.2
12	5	4.7	-0.3	5	4.4	-0.6	4	4.1	0.1
13	6	5.7	-0.3	6	5.4	-0.6	5	5.1	0.1
14	7	6.8	-0.2	6	6.6	0.6	5	6.3	1.3
15	5	5.7	0.7	5	5.4	0.4	5	5.1	0.1
16	5	4.7	-0.3	5	4.4	-0.6	4	4.1	0.1
17	6	5.7	-0.3	5	5.3	0.3	5	4.9	-0.1
18	6	5.7	-0.3	5	5.4	0.4	4	5.1	1.1
19	6	5.8	-0.2	6	5.3	-0.7	5	4.8	-0.2
20	6	5.7	-0.3	6	5.4	-0.6	5	5.1	0.1
21	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
22	4	3.6	-0.4	4	3.3	-0.7	3	3.0	0.0
23	5	4.6	-0.4	5	4.2	-0.8	4	3.8	-0.2
24	4	3.6	-0.4	4	3.2	-0.8	4	3.0	-1.0
25	4	3.6	-0.4	4	3.2	-0.8	4	3.0	-1.0
26	5	4.7	-0.3	5	4.3	-0.7	4	4.0	0.0
27	5	4.7	-0.3	4	4.4	0.4	4	4.1	0.1
28	4	3.6	-0.4	3	3.3	0.3	3	3.0	0.0
29	6	5.7	-0.3	6	5.4	-0.6	5	4.9	-0.1
30	6	5.7	-0.3	6	5.6	-0.4	5	5.1	0.1
31	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
32	6	5.7	-0.3	5	5.4	0.4	5	5.1	0.1
33	4	3.6	-0.4	4	3.3	-0.7	4	3.0	-1.0
34	4	3.6	-0.4	3	3.3	0.3	3	3.0	0.0
35	4	3.6	-0.4	3	3.3	0.3	3	3.0	0.0
36	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
37	4	3.6	-0.4	3	3.3	0.3	3	3.0	0.0
38	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
39	3	3.0	0.0	3	3.0	0.0	3	3.0	0.0
40	4	3.5	-0.5	4	3.0	-1.0	3	3.0	0.0

Figure 4: Condition Rating Prediction Errors (1999)

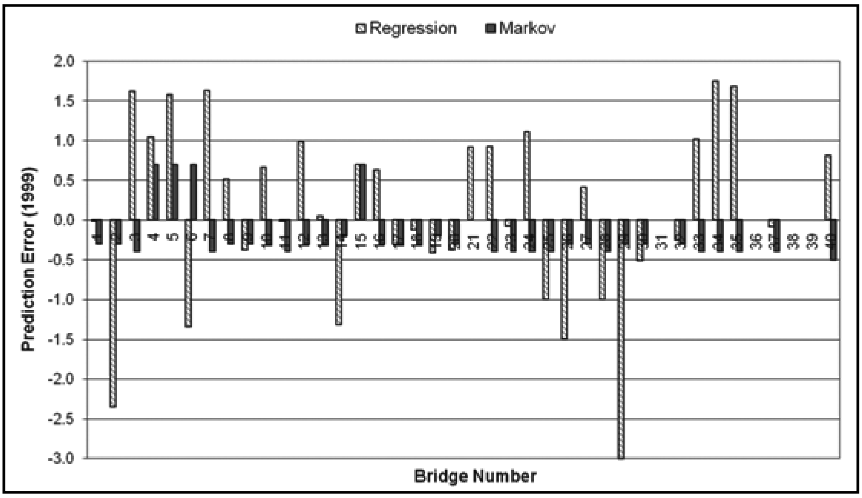


Figure 5: Condition Rating Prediction Errors (2001)

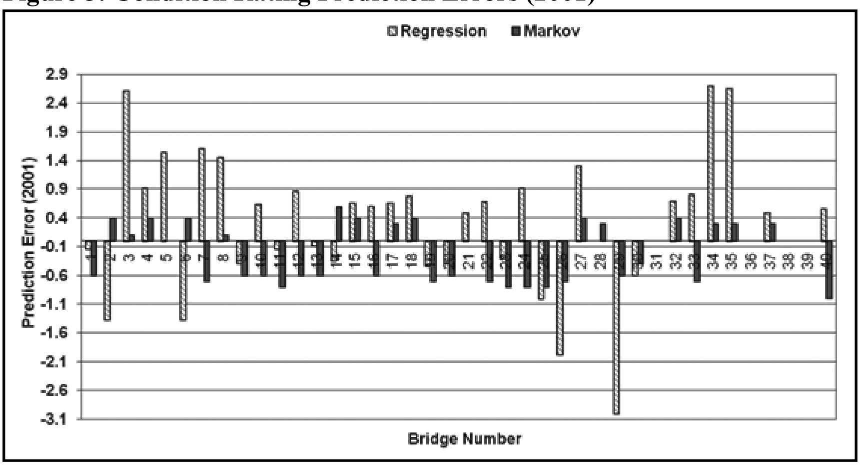


Figure 6: Condition Rating Prediction Errors (2003)

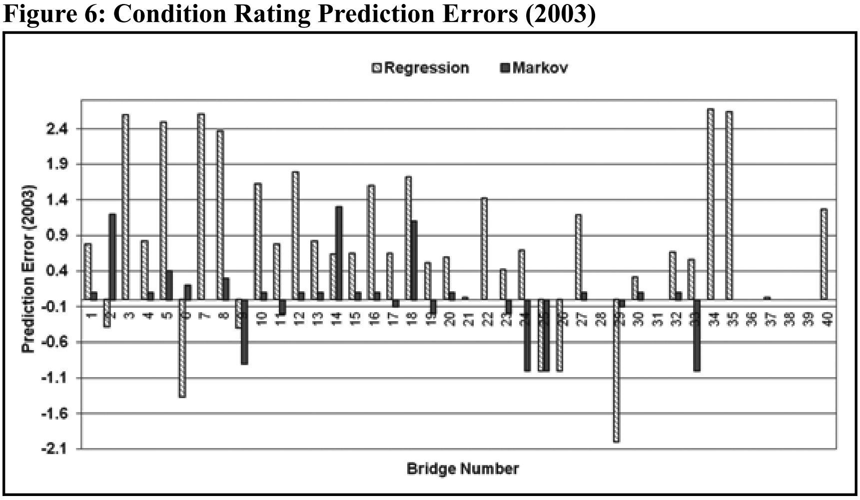


Figure 7: Average Prediction Errors

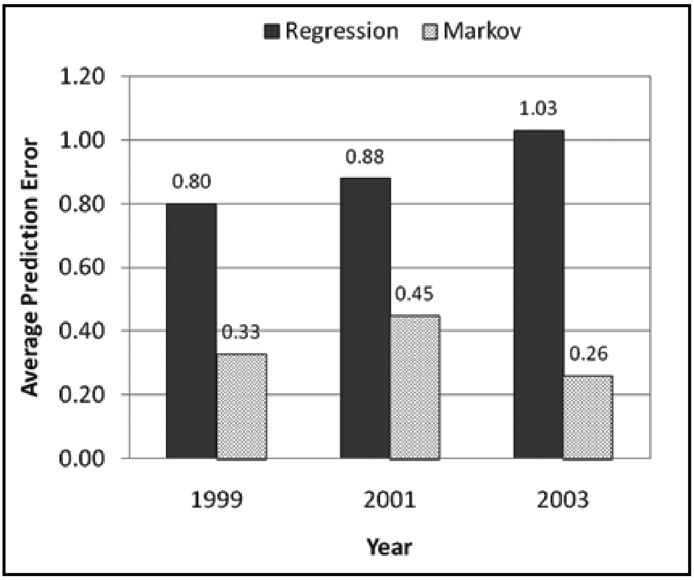
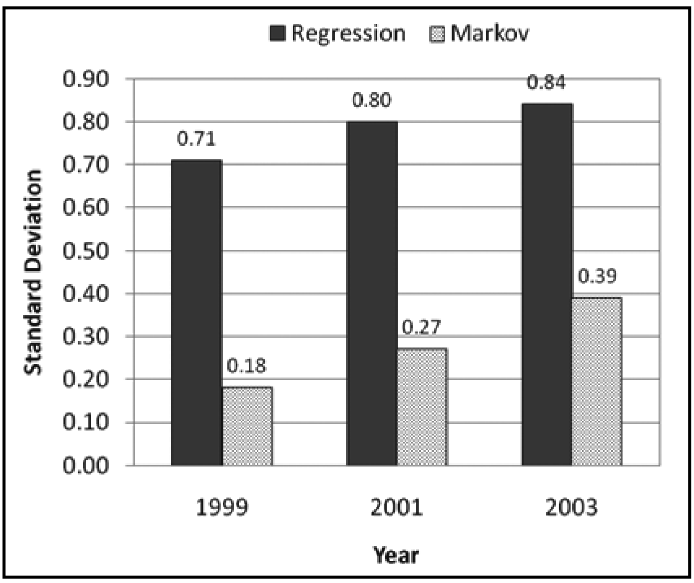


Figure 8: Standard Deviations of Prediction Errors



SYSTEM OPTIMIZATION

Since bridge project selection and system benefit optimization depend on bridge conditions, the accuracies of bridge condition predictions would certainly affect the optimization results. As shown in the integer linear programming optimization model (Equations 10 through 14), available budget also has major impact on project selections. To analyze the effects of condition predictions on bridge system optimization, the optimization program was run based on the actual condition ratings and the condition predictions from the two prediction models. In addition, to examine the effects of available budget, 40%, 80%, and 100% of needed budgets were used as input constraints of the integer linear programming.

The optimization program determines which bridges should be rehabilitated or replaced at each time period. The optimization results are presented in Table 4. As can be seen in the table, the selected bridge projects and the total expected benefits are different for actual, regression predicted, and Markov chain predicted condition ratings. With 100% needed budget, all of the 40 bridges are selected in the six-year period for each of the three sets of condition ratings. However, the sequences of the bridges to be rehabilitated or replaced are different. With sufficient budget, the total benefit values are 409,642, 417,476, and 413,805 for actual, regression predicted, and Markov chain predicted condition ratings, respectively. With 80% and 40% needed budgets, each optimization selects less than 40 bridges because of the insufficient amount of funds. As a result, the maximized system benefits under insufficient funds are also different for the three sets of condition ratings as shown in Table 4. The results in the table indicate that the total system benefits fall for all of the three conditions as the available budget decreases. The function of the optimization program is to maximize the total benefit based on the predicted bridge conditions with the limited budget.

The total benefits shown in Table 4 are plotted in Figure 9 to illustrate the benefit values based on the actual, regression predicted, and Markov predicted condition ratings. The figure indicates that, compared with the regression based total benefits, the Markov based total benefits are closer to the actual total benefits. If the total benefit based on the actual condition rating is called the "true benefit," the benefit deviation can be defined as the difference between the optimized total benefit and the true benefit. A positive benefit deviation represents an overestimate of system benefit and a negative value means an underestimate of system benefit. For example, with a 100% needed budget, the true benefit is 409,642, the benefit deviation for the polynomial regression predictions can be calculated as 417,476-409,642=7,834, and for the Markov chain predictions, 413,806-409,642=4,163. That is, the optimization based on regression predictions resulted in greater benefit deviation from the true benefit than the Markov chain predictions. The benefit deviations are shown in Figure 10. As depicted in Figure 10, as the budget increases, the magnitude of benefit deviation for each optimization decreases. In all cases, the optimizations based on the regression predictions generate greater magnitudes of benefit deviations than those based on the Markov chain predictions. In other words, the optimization based on Markov chain predictions would result in more accurate and more realistic system benefits and project selections.

Table 4: Project Selections and System Benefits with Different Available Budgets

Time	Optimization Results with 100% Needed Budget		
	Bridges Selected Using Actual Rating	**Bridges Selected Using Regression Predicted Rating**	**Bridges Selected Using Markov Predicted Rating**
1998, 1999	Bridge Numbers: 1, 17, 20, 24, 30, 34	Bridge Numbers: 1, 3, 5, 6, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 24, 25, 26, 31, 32, 33, 38, 39	Bridge Numbers: 1, 3, 5, 6, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 24, 25, 26, 31, 32, 33, 38, 39
2000, 2001	Bridge Numbers: 3, 5, 6, 7, 8, 10, 13, 14, 15, 16, 18, 19, 23, 25, 26, 31, 32, 33, 37, 38, 39	Bridge Numbers: 2, 10, 21, 23, 29, 34, 35	Bridge Numbers: 4, 10, 12, 21, 23, 29, 30, 35
2002, 2003	Bridge Numbers: 2, 4, 9, 11, 12, 21, 22, 27, 28, 29, 35, 36, 40	Bridge Numbers: 4, 9, 11, 12, 22, 27, 28, 30, 36, 37, 40	Bridge Numbers: 2, 9, 11, 22, 27, 28, 34, 36, 37, 40
	Total Benefit = 409,642	***Total Benefit = 417,476***	***Total Benefit = 413,805***
Time	**Optimization Results with 80% Needed Budget**		
	Bridges Selected Using Actual Rating	**Bridges Selected Using Regression Predicted Rating**	**Bridges Selected Using Markov Predicted Rating**
1998, 1999	Bridge Numbers: 6, 7, 13, 14, 15, 16, 19, 20, 24, 25, 26, 31, 32, 33, 38	Bridge Numbers: 6, 7, 13, 14, 15, 16, 19, 20, 24, 25, 26, 31, 32, 33, 38	Bridge Numbers: 7, 13, 14, 15, 16, 18, 19, 20, 24, 25, 26, 31, 32, 33, 38
2000, 2001	Bridge Numbers: 1, 3, 5, 8, 17, 23, 29, 30, 37, 39	Bridge Numbers: 1, 2, 3, 8, 17, 18, 23, 29, 37, 38	Bridge Numbers: 1, 3, 4, 5, 6, 8, 10, 11, 12, 17, 23, 29, 30, 34, 39
2002, 2003	Bridge Numbers: 2, 4, 9, 10, 11, 12, 21, 27, 28, 34, 35, 40	Bridge Numbers: 4, 5, 10, 11, 12, 21, 27, 28, 30, 34, 35, 39, 40	Bridge Numbers: 2, 21, 27, 28, 35, 37, 40
	Total Benefit = 391,904	***Total Benefit = 378,206***	***Total Benefit = 397,163***
Time	**Optimization Results with 40% Needed Budget**		
	Bridges Selected Using Actual Rating	**Bridges Selected Using Regression Predicted Rating**	**Bridges Selected Using Markov Predicted Rating**
1998, 1999	Bridge Numbers: 6, 7, 13, 14, 15, 16, 18, 19, 26, 31, 32, 33, 38	Bridge Numbers: 6, 13, 14, 15, 16, 18, 19, 20, 26, 31, 32, 33, 38, 39	Bridge Numbers: 6, 7, 13, 14, 15, 16, 18, 19, 26, 31, 32, 33, 38
2000, 2001	Bridge Numbers: 24, 25	Bridge Numbers: 24, 25	Bridge Numbers: 1, 3, 4, 8, 10, 12, 17, 20, 25, 30, 39
2002, 2003	Bridge Numbers: 1, 3, 9, 17, 20, 23, 30	Bridge Numbers: 1, 3, 7, 8, 23, 30	Bridge Numbers: 2, 5, 11, 24, 34, 35
	Total Benefit = 319,905	***Total Benefit =259,932***	***Total Benefit = 310,996***

Figure 9: System Benefits with Different Available Budgets

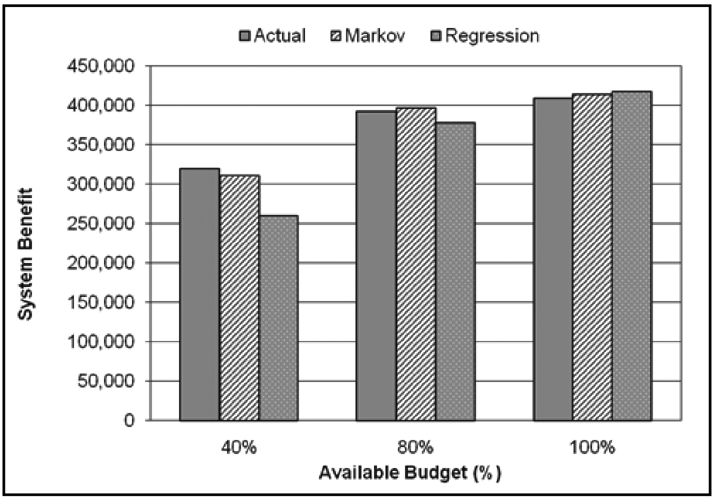
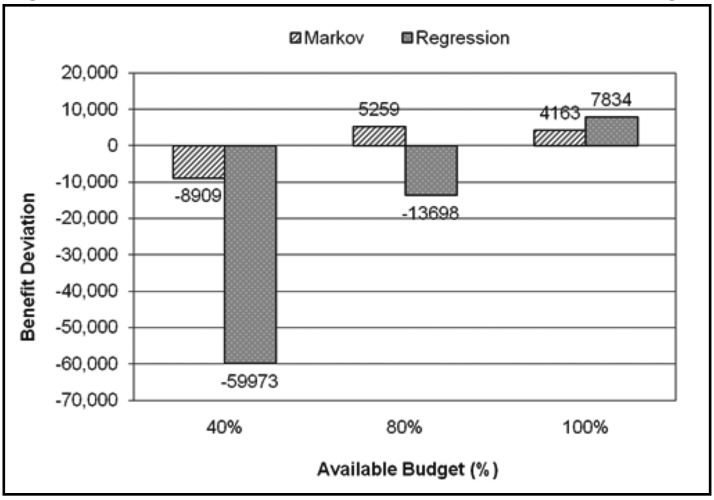


Figure 10: Benefit Deviations with Different Available Budgets



CONCLUSIONS

It is demonstrated that the Markov chain method yields more accurate condition predictions than the polynomial regression method. The accuracy of condition prediction affects the optimization results greatly, in terms the number of selected projects and their schedules as well as the total system benefit. A less accurate condition prediction will consequently produce a greater benefit deviation from the true system benefit. It is essential for a bridge management system to have a capability to provide highly accurate condition predictions. Otherwise, the optimization techniques would not provide meaningful results that would truly maximize the system benefits. Although this study focused on bridges, the study results and findings can also be applied in similar areas, such as pavement management systems and other transportation infrastructure management systems.

The results of this paper were obtained using Indiana bridge condition data. Although the author believes that the Markov method should produce better bridge condition predictions in general, the findings from this study should not be generalized without validation with bridge data from other states. Nonetheless, the analysis method discussed in this paper could be used by other highway agencies to choose a more accurate estimation technique.

References

AASHTO (American Association of State Highway and Transportation Officials). "Guide for Commonly Recognized (CoRe) Structural Elements." Washington, D.C., 1995.

Busa, G., M. Ben-Akiva, and O. Buyukozturk. "Modeling Concrete Deck Deterioration." Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1985.

Butt, A.A., K.J. Feighan, and M.Y. Shahin. "Pavement Performance Prediction Model Using the Markov Process." *Transportation Research Record 1123*, Transportation Research Board, Washington, D.C. (1987): 12-19.

Caleyo, F., J.C. Velázquez, A. Valor, and J.M. Hallen. "Markov Chain Modeling of Pitting Corrosion in Underground Pipelines." *Corrosion Science* 51 (9), (2009): 2197-2207.

Fitzpatrick, M., D. Law, and W. Dixon. "Deterioration of New York State Highway Structures." *Transportation Research Record 800*. Transportation Research Board, National Research Council, Washington, D.C. (1981): 1-8.

FHWA (Federal Highway Administration). "National Bridge Inspection Program: Assessment of FHWA's Implementation of Data-Driven, Risk-Based Oversight." Report Number: MH-2009-013, 2009.

Hymon, W., D. Hughes, and T. Dobson, "The Least Cost Mix of Bridge Replacement and Repair Work on Wisconsin's State Highways Over Time – a Computer Simulation." Technical Report, Wisconsin Department of Transportation, Madison, Wisconsin, 1983.

Jiang, Y. and K.C. Sinha. "The Development of Optimal Strategies for Maintenance, Rehabilitation and Replacement of Highway Bridges." Final Report Vol. 6: Performance Analysis and Optimization. FHWA/IN/JHRP-89/13, 1989.

Li, N., W. Xie, and R. Haas. "Reliability-Based Processing of Markov Chains for Modeling Pavement Network Deterioration." *Transportation Research Record 1524*, Transportation Research Board, National Research Council, Washington, D.C. (1996): 203-213.

Li, W. and C. Zhang. "A Random-Path Markov Chain Algorithm for Simulating Categorical Soil Variables from Random Point Samples." *Soil Science Society of America Journal* 71 (3), (2007): 656-668.

Micevski, T., G. Kuczera, and P. Coombes. "Markov Model for Storm Water Pipe Deterioration." *Journal of Infrastructure Systems* 8 (2), (2002): 49-56.

Neter, J., W. Wasserman, and M. Kutner. *Applied Linear Statistical Models: Regression, Analysis of Variance, and Experimental Designs.* Richard D. Irwin, Inc., Homewood, Illinois, 1985.

Sinha, K.C., M. Saito, Y. Jiang, S. Murthy, A.B. Tee, and M.D. Bowman. "The Development of Optimal Strategies for Maintenance, Rehabilitation and Replacement of Highway Bridges." Final Report Vol. 1: The Elements of Indiana Bridge Management System (IBMS). FHWA/IN/JHRP-88/15, 1989.

USDOT (U.S. Department of Transportation). "Status of the Nation's Highways, Bridges, and Transit: 2004 Conditions and Performance, Report to Congress." Washington, D.C., 2006.

Winston, W.L. Operations Research – Applications and Algorithms. Fourth Edition. Duxbury Press, Belmont, California. 2003.

Yang, C.W., P.Y. Kim, B.J. Ponsford, and B.C. Garland. "Impact of Wal-Mart on Market Share in a Rural Grocery Market: An Application of the Markov Chain Model." *Journal of Food Products Marketing* 16 (2), 2010: 232-245.

Yi Jiang *is an associate professor in the Department of Building Construction Management, Purdue University. He is a registered professional engineer in the state of Indiana. He received his bachelor's degree in civil engineering from Tongji University in China and his M.S. and Ph.D. degrees in civil engineering from Purdue University.*